

Fermi surface topology and the upper critical field in two-band superconductors - application to MgB₂

T. Dahm and N. Schopohl

*Institut für Theoretische Physik, Universität Tübingen,
Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

(Dated: February 1, 2008)

Recent measurements of the anisotropy of the upper critical field B_{c2} on MgB₂ single crystals have shown a puzzling strong temperature dependence. Here, we present a calculation of the upper critical field based on a detailed modeling of bandstructure calculations that takes into account both the unusual Fermi surface topology and the two gap nature of the superconducting order parameter. Our results show that the strong temperature dependence of the B_{c2} anisotropy can be understood as an interplay of the dominating gap on the σ -band, which possesses a small c -axis component of the Fermi velocity, with the induced superconductivity on the π -band possessing a large c -axis component of the Fermi velocity. We provide analytic formulas for the anisotropy ratio at $T = 0$ and $T = T_c$ and quantitatively predict the distortion of the vortex lattice based on our calculations.

PACS numbers: 74.20.-z, 74.25.Op, 74.70.Ad

Our understanding of the physical properties of the recently discovered superconductivity in MgB₂ has made rapid progress since its discovery [1]. Its high critical temperature $T_c = 39$ K can be understood as arising from strong conventional electron-phonon coupling to a high frequency phonon mode [2, 3, 4]. Its pairing symmetry seems to be of conventional s -wave type [5, 6]. However, in contrast to conventional superconductors, a number of recent experiments indicate that there exist two gaps of different size in this compound [7, 8, 9, 10, 11, 12]. This possibility is supported by band structure calculations, which have shown that the Fermi surface of this compound consists of four bands: two σ -type two-dimensional cylindrical hole sheets and two π -type three dimensional tubular networks [13, 14] in good overall agreement with recent de Haas-van Alphen experiments [15]. Microscopic calculations of the superconducting gap based on band structure calculations have shown recently that indeed one should expect a big superconducting gap living on the σ bands and a smaller one, induced by interband pairing interaction, living on the π bands [3, 16]. Impurity scattering, which in conventional superconductors tends to average out strongly differing gap values, in this case becomes ineffective, because the σ and π bands possess different symmetries, making interband impurity scattering much weaker than intraband impurity scattering [17].

Recent measurements of the upper critical field B_{c2} , particularly its anisotropy, on single crystal MgB₂ have shown a puzzling strong temperature dependence of the anisotropy ratio B_{c2}^{ab}/B_{c2}^c between the ab-plane and the c-axis upper critical field [18, 19, 20]. In conventional systems this ratio rarely changes by more than 10 to 20 percent as a function of temperature. In MgB₂ changes by more than a factor of 2 have been observed. It has been shown that such a strong temperature dependence of the anisotropy ratio can be obtained within a strongly

anisotropic single gap model, possessing a small gap in c-axis direction and a big gap in ab-plane direction [21]. However, the gap anisotropy would have to be a factor of 10, which is too big as compared with experimental values. In addition, this scenario would be inconsistent with penetration depth studies which clearly indicate the presence of a small gap within the ab-plane [22, 23].

Here, we want to present first calculations of the upper critical field B_{c2} , which take into account the multi-band Fermi surface structure seriously. We show that the strong temperature dependence of the anisotropy of B_{c2} can be traced back to the influence of the two topologically very different Fermi surface types. While the cylindrical Fermi surface sheets are dominating the behavior of B_{c2} at low temperatures, leading to a large anisotropy, at temperatures approaching T_c the π bands due to their much larger c-axis Fermi velocity start to play a more important role, strongly reducing the B_{c2} anisotropy. Thus, the strong temperature dependence of the B_{c2} anisotropy appears as a cross-over from a low temperature σ band dominated regime to a higher temperature mixed σ and π band regime.

In order to include the multi-band Fermi surface structure in the calculation of the upper critical field, we start our investigation from the fully momentum dependent multi-band formulation of the quasiclassical (Eilenberger) theory of the upper critical field [24]. For that purpose we have to solve the linearized multi-band gap equation in the presence of an external magnetic field, which reads

$$\Delta_\alpha(\vec{r}) = -\pi T \sum_{\alpha'} \sum_{|\omega'_n| < \omega_c} \lambda^{\alpha\alpha'} \left\langle f_{\alpha'}(\vec{r}, \hat{k}'; \omega'_n) \right\rangle_{\alpha'} \quad (1)$$

Here, f_α is the anomalous Eilenberger propagator on the Fermi surface sheet denoted by α . Δ_α is the gap function on that Fermi surface sheet and $\lambda^{\alpha\alpha'}$ is the pairing interaction, which becomes a matrix in the band indices. The

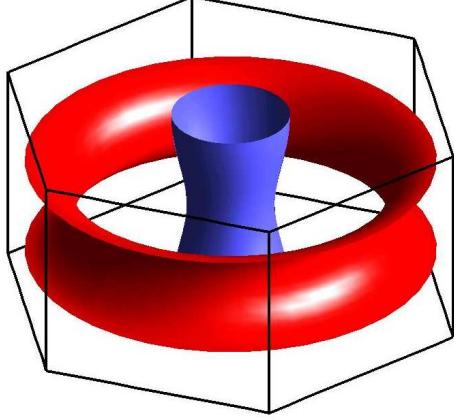


FIG. 1: (color). Fermi surface topology used for the calculation of B_{c2} in this work. The π -band is modeled by a half-torus, the σ -band by a distorted cylinder.

brackets $\langle \dots \rangle_{\alpha'}$ denote a Fermi surface average over momentum \hat{k}' of the Fermi surface sheet α' . In Eq. (1) we have already assumed that the gaps are isotropic s -wave on the Fermi surfaces, as indicated by experiment, but may have different values on different sheets. At B_{c2} the anomalous Eilenberger propagator has to be determined from the linearized Eilenberger equation (for $\omega_n > 0$)

$$\left\{ \omega_n + \vec{v}_{F,\alpha} \left[\frac{\hbar}{2} \vec{\nabla} - i \frac{e}{c} \vec{A}(\vec{r}) \right] \right\} f_\alpha(\vec{r}, \hat{k}; \omega_n) = -\Delta_\alpha(\vec{r}) \quad (2)$$

Here, $\vec{v}_{F,\alpha}$ is the (momentum dependent) Fermi velocity on Fermi surface sheet α and \vec{A} the vector potential due to the magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$. Eqs. (1) and (2) constitute an eigenvalue problem for $\Delta_\alpha(\vec{r})$. For a given temperature T the solution $\Delta_\alpha(\vec{r})$ which solves Eqs. (1) and (2) for the highest value of B determines B_{c2} .

Usually this eigenvalue problem is solved by a Landau level expansion of $\Delta_\alpha(\vec{r})$ above the Abrikosov ground state of the vortex lattice. It has been shown recently, however, that a variational ansatz for $\Delta_\alpha(\vec{r})$ corresponding to a *distorted* Abrikosov lattice leads to much better results for strongly anisotropic systems [21] and we will adopt that method here. In this method the ansatz reads $\Delta_\alpha(\vec{r}) = \Delta_\alpha \psi_\Lambda^\tau(\vec{r})$ with $\psi_\Lambda^\tau(x, y) = \psi_\Lambda(e^{-\tau}x, e^\tau y)$. Here, ψ_Λ is the usual Abrikosov groundstate and τ is a variational parameter describing the distortion of the vortex lattice. The undistorted lattice corresponds to $\tau = 0$ and τ has to be determined by maximizing B_{c2} . Introducing this ansatz into Eqs. (1) and (2) and using standard operator techniques (see e.g. Refs. [24, 25, 26, 27]) we are led to the following eigenvalue problem for Δ_α in band space:

$$\Delta_\alpha = \sum_{\alpha'} \lambda^{\alpha\alpha'} \left[\frac{1}{\lambda_+} - \ln \frac{T}{T_c} - l_{\alpha'}(\tau, \frac{B_{c2}}{T^2}) \right] \Delta_{\alpha'} \quad (3)$$

Here, λ_+ is the highest eigenvalue of $\lambda^{\alpha\alpha'}$, which determines T_c . The function l_α is given by the expression

$$l_\alpha = \int_0^\infty \frac{du}{\sinh u} \left\langle 1 - e^{-u^2 \frac{e B_{c2}}{8\pi^2 T^2} (e^{-2\tau} v_{F1,\alpha}^2(\hat{k}) + e^{2\tau} v_{F2,\alpha}^2(\hat{k}))} \right\rangle_\alpha \quad (4)$$

Here, $v_{F1,\alpha}$ and $v_{F2,\alpha}$ are the components of the Fermi velocity perpendicular to the magnetic field \vec{B} on Fermi surface α . According to Eq. (3) the criterion for B_{c2} is that the highest eigenvalue of the matrix $\lambda^{\alpha\alpha'} \left[\frac{1}{\lambda_+} - \ln \frac{T}{T_c} - l_{\alpha'}(\tau, \frac{B_{c2}}{T^2}) \right]$ becomes 1. Apparently, at $T = T_c$ this is fulfilled for $B_{c2} = 0$, because $l_\alpha \rightarrow 0$.

Eqs. (3) and (4) allow to determine the temperature dependence and angular dependence of B_{c2} from microscopic grounds. The material parameters we need for the solution are the Fermi velocities $\vec{v}_{F,\alpha}(\hat{k})$ and the coupling matrix $\lambda^{\alpha\alpha'}$, which can be taken from bandstructure calculations. In order to simplify the analysis we restrict ourselves to two relevant bands, because the two σ -bands and the two π -bands are very similar [3]. The σ -band can be described to a good approximation by a cylindrical Fermi surface with a small c -axis hopping parameter. The π -band can be modeled by a half-torus as shown in Fig. 1 (for comparison see the Γ -point centered Fermi surfaces obtained from bandstructure calculations in Fig. 1 in Ref. [13]). For the Fermi velocities on these two Fermi surfaces we thus write $\vec{v}_{F,\pi}(\theta, \phi) = v_{F\pi}(\cos \theta \cos \phi, \cos \theta \sin \phi, \sin \theta)$ and $\vec{v}_{F,\sigma}(k_c, \phi) = v_{F\sigma}(\cos \phi, \sin \phi, \epsilon_c \sin ck_c)$. Here, ϕ is the azimuthal angle within the ab -plane, $\theta \in [\frac{\pi}{2}, \frac{3\pi}{2}]$ the polar angle of the torus, k_c the c -axis component of the momentum, and c the lattice constant in c -direction. The dimensionless parameter ϵ_c describes the small c -axis dispersion of the cylinder. The Fermi surface averages over the cylinder and the torus are then given by $\langle \dots \rangle_\sigma = \frac{c}{4\pi^2} \int_{-\pi/c}^{\pi/c} dk_c \int_0^{2\pi} d\phi \dots$ and $\langle \dots \rangle_\pi = \frac{1}{2\pi^2} \int_{\pi/2}^{3\pi/2} d\theta \int_0^{2\pi} d\phi \frac{1+\kappa \cos \theta}{1-2\kappa/\pi} \dots$, where κ is the ratio of the two radii of the torus. The parameters of the Fermi velocities can be found from bandstructure calculations. Taking the values given in Ref. [28], we determine $v_{F\pi} = 8.2 \cdot 10^5 \text{ m/s}$, $v_{F\sigma} = 4.4 \cdot 10^5 \text{ m/s}$, and $\epsilon_c = 0.23$. The ratio of the two radii of the torus can be estimated from the Fermi surface crossings in Ref. [14] to be about $\kappa = 0.25$.

For a 2×2 matrix $\lambda^{\alpha\alpha'}$ the criterion that the biggest eigenvalue of Eq. (3) becomes 1 leads to the equation $(1 - \eta)l_\sigma + \eta l_\pi + \ln t = -\Lambda_\pm(l_\sigma + \ln t)(l_\pi + \ln t)$. Here, $t = T/T_c$, λ_- is the smaller eigenvalue of $\lambda^{\alpha\alpha'}$ and $\eta = \frac{\lambda^{\pi\pi} - \lambda_-}{\lambda_+ - \lambda_-}$ is a dimensionless parameter describing the interband coupling strength of the two bands ($\eta = 0$ corresponds to no coupling, $\eta = 0.5$ to maximum coupling). From bandstructure calculations in Ref. [3] the following effective matrix elements can be obtained: $\lambda^{\sigma\sigma} = 0.959$, $\lambda^{\sigma\pi} = 0.222$, $\lambda^{\pi\sigma} = 0.163$, and $\lambda^{\pi\pi} = 0.278$. From these we find $\lambda_+ = 1.008$, $\lambda_- = 0.228$, and $\eta = 0.064$. As it turns out the remaining parameter $\Lambda_\pm = \frac{\lambda_+ + \lambda_-}{\lambda_+ - \lambda_-}$ only

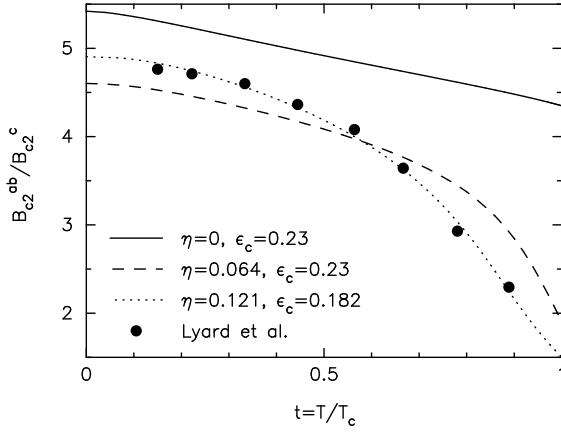


FIG. 2: Temperature dependence of the anisotropy ratio $\Gamma = B_{c2}^{ab}/B_{c2}^c$ for the two-band model described in the text and different interband coupling strengths η . Solid circles are experimental results taken from Lyard et al.[20].

weakly affects the results as soon as λ_- is sufficiently smaller than λ_+ , as is the case here.

We have calculated B_{c2} numerically for the material parameters above optimizing the parameter τ such that B_{c2} is maximized. In Fig. 2 we show our result for the anisotropy ratio $\Gamma = B_{c2}^{ab}/B_{c2}^c$ as a function of temperature for different values of the interband coupling strength η . For $\eta = 0$, when there is no coupling between the two bands, the temperature dependence of Γ is determined by the cylindrical σ -band because of its stronger pairing interaction. Here, Γ changes only by 20 percent, as one expects for an isotropic single gap superconductor. When η is increased, however, the temperature dependence of Γ becomes more pronounced. Our result for the parameters given above is shown as the dashed line. Note, that our calculation is parameter free, relying only on the parameters given by bandstructure calculations. For comparison, also the experimental data by Lyard et al. [20] are shown (solid circles). As will become clear below, the most important parameters determining the temperature dependence of Γ are the interband coupling strength η and the c -axis dispersion parameter ϵ_c . If we allow these two parameters to vary somewhat, we can obtain an excellent fit of the experimental data. The dotted line shows our result for $\eta = 0.121$ and $\epsilon_c = 0.182$, where the other parameters have been kept constant. We mention here that this set of parameters also gives a correspondingly good fit of the temperature dependences of B_{c2}^c and B_{c2}^{ab} separately including the upward curvature of B_{c2}^{ab} that has been noted in the experiments [29]. This is just an immediate consequence of the strong temperature dependence of the anisotropy ratio and B_{c2}^c varying linearly near T_c because of an absence of vortex lattice distortion in this field direction.

Apparently, the anisotropy ratio Γ turns out to be more sensitive to interband coupling η close to T_c than

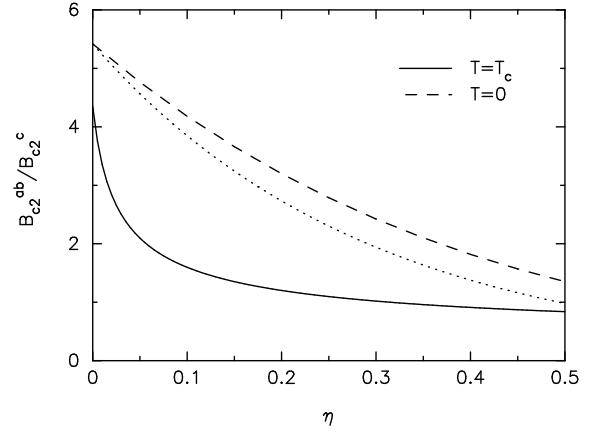


FIG. 3: Anisotropy ratio $\Gamma = B_{c2}^{ab}/B_{c2}^c$ as a function of interband coupling strength η for $T = 0$ (dashed line) and $T = T_c$ (solid line). The dotted line shows the approximation for $T = 0$ given in Eq. (7). At T_c the anisotropy ratio is much more sensitive to small interband coupling strengths η .

at $T = 0$. In order to get a better physical understanding for this behavior, we want to discuss some limits, in which analytical solutions for B_{c2} can be obtained. At first we consider the limit $T \rightarrow T_c$. In this case Eqs. (3) and (4) can be solved exactly and we obtain for the angular dependence of B_{c2} as a function of the angle β the magnetic field makes with the c -axis:

$$\frac{B_{c2}(\beta)}{B_{c2}(\beta=0)} = (\cos^2 \beta + A \sin^2 \beta)^{-1/2} \quad (5)$$

where $A = 2 \frac{(1-\eta)\langle v_{c,\sigma}^2 \rangle_\sigma + \eta \langle v_{c,\pi}^2 \rangle_\pi}{(1-\eta)\langle v_{ab,\sigma}^2 \rangle_\sigma + \eta \langle v_{ab,\pi}^2 \rangle_\pi}$. Here, $\langle v_{c,\alpha}^2 \rangle_\alpha$ is the average of the squared c -axis component of the Fermi velocity over Fermi surface α , while $\langle v_{ab,\alpha}^2 \rangle_\alpha$ is the corresponding in-plane quantity. The distortion parameter τ is given by $e^{-2\tau} = \frac{B_{c2}(\beta)}{B_{c2}(\beta=0)}$. When we introduce the numerical values for $v_{F\pi}$, $v_{F\sigma}$ and κ given above we find for the anisotropy ratio

$$\frac{B_{c2}^{ab}}{B_{c2}^c} = \frac{1}{\sqrt{A}} = \frac{1}{\epsilon_c} \sqrt{\frac{1 + 0.63\eta}{1 + \eta \left(\frac{3.69}{\epsilon_c^2} - 1 \right)}} \quad (6)$$

For $\epsilon_c = 0.23$ this result is shown in Fig. 3 as a function of η (solid line). From Eq. (6) we see that at $\eta = 0$ the anisotropy is given by the c -axis dispersion of the cylindrical Fermi surface and actually diverges for $\epsilon_c \rightarrow 0$. (Note, that this divergence could not have been obtained from a Landau level expansion above the Abrikosov ground state and our variational ansatz above is crucial for this result). Once the interband coupling η is increased, the anisotropy quickly reduces. This reduction becomes sizeable already, when $\eta \sim \frac{\langle v_{c,\sigma}^2 \rangle_\sigma}{\langle v_{c,\pi}^2 \rangle_\pi} = \frac{\epsilon_c^2}{3.69}$. Thus, it is the small c -axis Fermi velocity of the cylindrical Fermi surface as compared to the one of the π -band

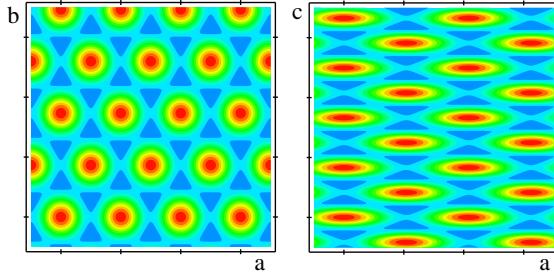


FIG. 4: (color). Vortex lattice structure for magnetic field in c -axis direction (left panel) and in ab -plane direction (right panel) at zero temperature calculated from the two-band model described in the text. For comparison, the area of the unit cell has been kept fixed.

which leads to a high sensitivity of Γ near T_c and a small interband coupling is already sufficient to make the influence of the π -band visible.

In the limit $T \rightarrow 0$ the integration in Eq. (4) can be performed and we find $l_\alpha(\tau, \frac{B_{c2}}{T^2}) = \frac{1}{2} \ln \left(\frac{\gamma e B_{c2}}{2\pi^2 T^2} \right) + \frac{1}{2} \langle \ln (e^{-2\tau} v_{F1,\alpha}^2 + e^{2\tau} v_{F2,\alpha}^2) \rangle_\alpha$, where $\ln \gamma = 0.577$ is Euler's constant. Since the pairing interaction is dominant in the σ -band, we can employ two approximations: at first we set $\lambda_- = 0$. This corresponds to assuming that superconductivity in the π -band is completely induced by interband coupling. As a second approximation we assume that the vortex lattice distortion τ is also dominated by the σ -band. Then we can find τ by just optimizing it at $\eta = 0$ and then use it for evaluation at $\eta > 0$. With these two approximations the logarithmic averages in l_α can be performed and we finally get

$$\frac{B_{c2}^{ab}(T=0)}{B_{c2}^c(T=0)} = \frac{1.246}{\epsilon_c} e^{-\eta(0.482 - 2\ln \epsilon_c)} \quad (7)$$

The parameter τ is found to be $\tau = \frac{1}{2} \ln \epsilon_c$, which shows that at low temperatures there is no simple relation anymore between τ and the anisotropy ratio. Also we mention here that at low temperatures Eq. (5) does not hold anymore. In Fig. 3 Eq. (7) is shown for $\epsilon_c = 0.23$ as the dotted line. In order to demonstrate the quality of these approximations also the fully numerical result without these approximations is shown as the dashed line. Apparently, now the anisotropy Γ is much less sensitive to interband coupling and only varies on a scale given by $\eta \sim 1/(0.482 - 2\ln \epsilon_c)$. The reason for this is that ϵ_c now only comes in logarithmically.

Eq. (7) shows that at $T = 0$ the divergence for $\epsilon_c \rightarrow 0$ appears at all $\eta < 0.5$, while at $T = T_c$ this divergence only shows up at $\eta = 0$. Numerically we observe that at finite η and $\epsilon_c = 0$ there is a certain temperature at which B_{c2}^{ab} diverges. This temperature becomes smaller when η is increased. In Fig. 4 we show the distortion of the vortex lattice at high magnetic field and low temperature that we expect from our calculation. When the

magnetic field is directed along the c -axis of the crystal a regular Abrikosov lattice is expected as shown in Fig. 4 (left panel). However, when the field is directed within the ab -plane we expect a distortion of $e^\tau = \sqrt{\epsilon_c} = 0.48$ as shown in Fig. 4 (right panel). This prediction can be checked by neutron scattering or STM tunneling [30].

To summarize, we have calculated the anisotropy of the upper critical field for the two band Fermi surface topology shown in Fig. 1. Using parameters from band-structure calculations for MgB₂ we find a strong temperature dependence of the upper critical field in agreement with recent measurements on MgB₂ single crystals. Fine tuning of the parameters can yield a very good fit of the experimental data. We observe that the small c -axis dispersion of the σ -band leads to a high sensitivity of the anisotropy ratio on the interband coupling strength near T_c , but not at $T = 0$. This suggests that the interplay of these two quantities leads to the strong temperature dependence of the upper critical field in MgB₂.

We would like to thank F. Bouquet, O.V. Dolgov, M.R. Eskildsen, K. Maki, A.I. Posazhennikova and L. Tewordt for valuable discussions. Thanks are also due to S. Graser for his help.

- [1] J. Nagamatsu et al, Nature(London) **410**, 63 (2001).
- [2] Y. Kong et al, Phys. Rev. B **64**, 020501(R) (2001).
- [3] A. Y. Liu, I. I. Mazin, and J. Kortus, Phys. Rev. Lett. **87**, 087005 (2001).
- [4] K.-P. Bohnen et al, Phys. Rev. Lett. **86**, 5771 (2001).
- [5] F. Manzano et al, Phys. Rev. Lett. **88**, 047002 (2001).
- [6] H. D. Yang et al, Phys. Rev. Lett. **87**, 167003 (2001).
- [7] P. Szabo et al, Phys. Rev. Lett. **87**, 137005 (2001).
- [8] F. Giubileo et al, Phys. Rev. Lett. **87**, 177008 (2001).
- [9] F. Bouquet et al, Europhys. Lett. **56**, 856 (2001).
- [10] A. V. Sologubenko et al, Phys. Rev. B **66**, 014504 (2002).
- [11] F. Bouquet et al, Phys. Rev. Lett. **89**, 257001 (2002).
- [12] M. Iavarone et al, Phys. Rev. Lett. **89**, 187002 (2002).
- [13] S. V. Shulga et al, cond-mat/0103154.
- [14] J. Kortus et al, Phys. Rev. Lett. **86**, 4656 (2001).
- [15] A. Carrington et al, cond-mat/0304435.
- [16] H. J. Choi et al, Nature(London) **418**, 758 (2002).
- [17] I. I. Mazin et al, Phys. Rev. Lett. **89**, 107002 (2001).
- [18] M. Angst et al, Phys. Rev. Lett. **88**, 167004 (2002).
- [19] Yu. Eltsev et al, Phys. Rev. B **65**, 140501(R) (2002); Physica C **378-381**, 61 (2002).
- [20] L. Lyard et al, Phys. Rev. B **66**, 180502(R) (2003).
- [21] A. I. Posazhennikova, T. Dahm, and K. Maki, Europhys. Lett. **60**, 134 (2002).
- [22] B. B. Jin et al, Phys. Rev. B **66**, 104521 (2002).
- [23] T. Dahm et al, Acta Physica Polonica B **34**, 549 (2003).
- [24] For a review see C.T. Rieck, K. Scharnberg, and N. Schopohl, J. Low Temp. Phys. **84**, 381 (1991).
- [25] N. Schopohl, J. Low Temp. Phys. **41**, 409 (1980).
- [26] T. Dahm, S. Graser, C. Iniotakis, and N. Schopohl, Phys. Rev. B **66**, 144515 (2002).
- [27] Y. Sun and K. Maki, Phys. Rev. B **47**, 9108 (1993).
- [28] A. Brinkman et al, Phys. Rev. B **65**, 180517(R) (2002).
- [29] T. Dahm et al, cond-mat/0304194.

[30] M. R. Eskildsen et al, Phys. Rev. Lett. **89**, 187003 (2002).